Continuous wave solid state laser systems with intra-cavity second harmonic generation [1–4]

PACS numbers:

I. FUNDAMENTALS

We will at first recall the key aspects of the laser process. The basic principle of amplification of a light wave transmitting through a laser medium is shown in Fig. 1, where u_{in} and u_{out} denote the incoming and outgoing photon flux of the light wave with $u_{out} >> u_{in}$. The phenomenon of amplification and its efficiency result from light interaction processes with the laser medium, which are summarized as follows.

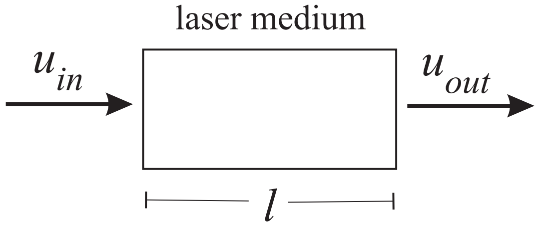

FIG. 1: Basic Principle of light amplification. u_{in} and u_{out} denote the incoming and outgoing flux of the light wave with $u_{out} >> u_{in}$

A. Absorption

Resonant excitation of electrons from the ground to an excited atomic state of the laser medium occurs if the energy of the incoming photon $E_{ph} = \hbar\omega$ reaches the energy difference between both states $E_{ph} = \Delta E = E_2 - E_1$ (Fig. 2a). Here, $N_{1,2}$ denote the number of atoms in the energy state $E_{1,2}$ per cm^3. The intensity of the transmitted light wave decreases, i.e., absorption occurs at the resonance frequency ω_0 with a finite full width at half maximum of the absorption band $\Delta\nu$ (Fig. 2b). The number of absorbed photons is described by:

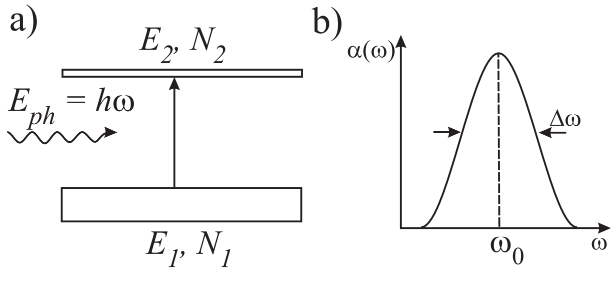

FIG. 2: a) Energy model of the absorption process. Resonant excitation of electrons from the ground to an excited atomic state occurs if the energy of the incoming photon $E_{ph} = \hbar\omega$ reaches the energy difference between both states $E_{ph} = \Delta E = E_2 - E_1$. b) Absorption band centered at the resonance frequency ω with the full width at half maximum $\Delta\nu$.

$$Z_a = N_1 \cdot u_{in} \cdot B_{12} \cdot f(\omega) \quad (1)$$

with the Einstein (or probability) coefficient B_{12} and the function $f(\omega)$ which takes the frequency dependence into account. The number of transmitted photons Z_t is thereby connected to the number of incoming photons Z_0 via $Z_t = Z_0 - Z_a$.

B. Spontaneous emission

Assuming a finite number of atoms in the electronic state E_2, i.e., $N_2 \neq 0$, the process of spontaneous emission occurs. It is a result of the limited lifetime of excited atoms, which is reciprocally proportional to the bandwidth of the absorption band $\tau \sim 1/\Delta\omega$. Typical values are $\tau \sim 10^{-8}$s. The transition of atoms $E_2 \rightarrow E_1$, and thus $N_2 \rightarrow N_1$, is accompanied by the emission of a photon with energy ΔE. A characteristic feature of this process is the emission of photons into all directions of space. The number of spontaneously emitted photons is described via $Z_s = N_2 \cdot A$ with the Einstein coefficient $A \sim 1/\tau$. The fraction of the Einstein coefficients of the absorption and the spontaneous emission is given by:

$$\frac{A}{B_{12}} = \frac{2}{\pi} \frac{\hbar\omega}{c^3} \quad (2)$$

with c the velocity of light in vacuum.

C. Induced emission

Induced emission occurs if there is a finite number of atoms in the electronic state E_2, i.e., $N_2 \neq 0$, and a resonant photon is present. In this case a photon $E_{ph}^{ie} = \Delta E$ is emitted. In contrast to spontaneous emission the induced emission of a photon occurs in the same direction as the incoming photon. Thus the photon flux of the incoming wave can be amplified:

$$Z_t = Z_0 + Z_i = Z_0 + N_2 \cdot U_{in} \cdot B_{21} \cdot f(\omega) \quad (3)$$

The energetic balance of an incoming photon flux to a laser medium with $N_1, N_2 \neq 0$ thus results to:

$$Z_t = Z_0 + Z_i - Z_a = \Delta N \cdot u_{in} \cdot B_{12} \cdot f(\omega) \quad (4)$$

with $\Delta N = N_2 - N_1$. There cases are distinguished:

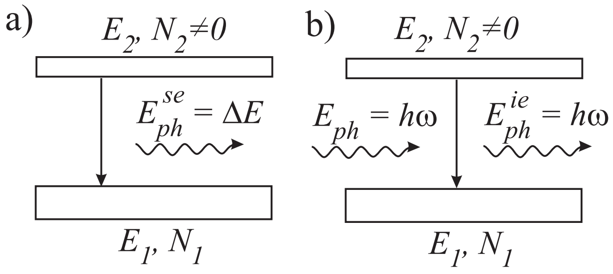

FIG. 3: a) Spontaneous emissions of a photon $E_{ph}^{se} = \Delta E$ due to relaxation processes of excited atoms. b) Induced emission of a photon $E_{ph}^{ie} = \Delta E$ by an incoming photon. In both cases $N_2 \neq 0$ is required.

- $N_2 < N_1 : \Delta N < 0$, i.e., depletion of the incoming light wave
- $N_2 = N_1 : \Delta N = 0$, i.e., unaffected transmission of the light wave
- $N_2 > N_1 : \Delta N > 0$, i.e., amplification of the incoming light wave.

The latter case is commonly denoted as occupation inversion. In the thermal equilibrium the occupation of the ground and excited states as a function of the temperature T follows the connection $N_2 = N_1 \exp(-\Delta E/k_B T)$, with the Boltzmann constant k_B. Note that $N_2 \approx 0$ at room temperature since $k_B T << \Delta E$. For very high temperatures $N_2 = N_1$ is reached resulting in an unaffected transmission of a light wave through the laser beam. Thus an occupation inversion can not be realized in the thermal equilibrium at any temperature.

Possibilities to reach occupation inversion are given by laser media with an energetic 3- or 4-level system.

D. 3-level system

The energetic scheme of a 3-level system, e.g., a ruby laser, is shown in Fig 4.

An occupation inversion $\Delta N = N_2 - N_1$ is reached under intense illumination with light of $E_{ph}^{p} = E_3 - E_1$, so that light of $E_{ph} = E_2 - E_1$ can be amplified. Population of N_2 results via de-excitation of atomic states $E_3 \rightarrow E_2$, whereby E_3 is optically excited. Thus this process is commonly called optical pumping. However, the population of each state and especially the occupation inversion is very sensitive to the intensity W of the pump light as shown in Fig 5.

Specific situations of the population ratio can be distinguished for the following intensities:

- $W = 0 : N_2 = 0 \Rightarrow \Delta N/N_0 = -1$
- $W < W_0 : N_1 > N_2 \Rightarrow \Delta N/N_0 < 0$
- $W = W_0 : N_1 = N_2 \Rightarrow \Delta N/N_0 = 0$

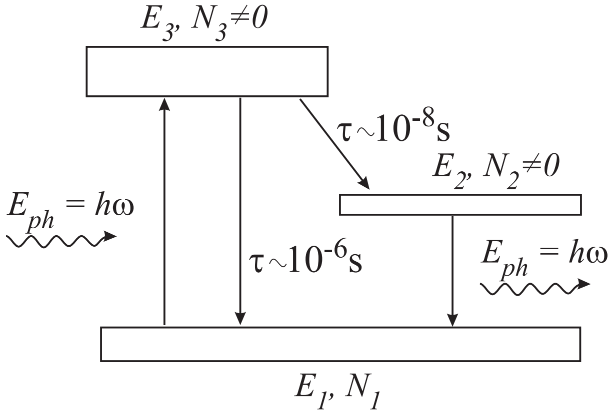

FIG. 4: energetic scheme of a 3-level system.

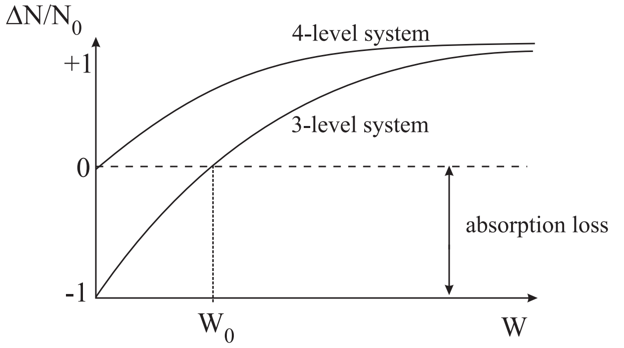

FIG. 5: Dependence of the ratio $\Delta N/N_0$ on the intensity of the pump light for a 3- and 4-level system.

- $W > W_0 : N_2 > N_1 \Rightarrow \Delta N/N_0 > 0$
- $W >> W_0 : N_1 \approx 0 \Rightarrow \Delta N/N_0 = 1$

It is obvious that the occupation inversion $\Delta N/N_0 > 0$ occurs for intensities higher than the threshold intensity W_0. Absorption dominates the transmission of the light wave for intensities $< W_0$.

E. 4-level system

The scheme of a 4-level system, e.g., Nd-YAG laser, is shown in Fig. 6. The key feature of the 4-level system is that E_1 is empty in the thermal equilibrium, i.e., occupation inversion is present as soon as $N_2 \neq 0$. This feature is connected with the comparable small lifetime of the atomic states in E_1. As a result there is no threshold behavior of the ratio $\Delta N/N_0$ on the intensity as shown in Fig. 5.

The efficiency of amplification further depends on the interaction length of the light wave in the laser media by:

$$I_{out} = I_{in} \exp\left(\frac{B_{12}\Delta N}{c} \cdot l\right) \qquad (5)$$

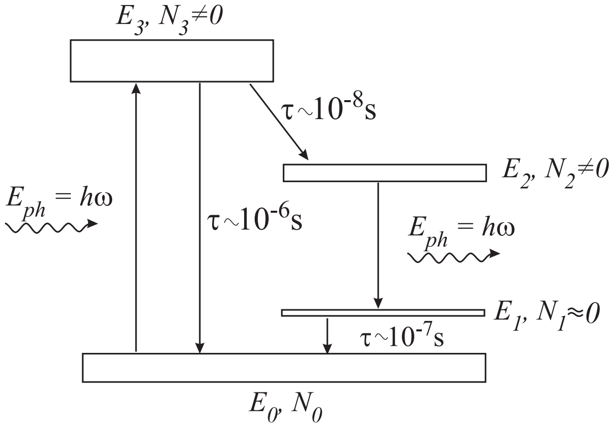

FIG. 6: energetic scheme of a 4-level system.

It should be noted, that N_2 reaches saturation with increasing intensity of the amplified light wave and that there is a non-linear dependence of the amplified intensity on the pump intensity as well as interaction length. the gain $\Gamma = I_{out}/I_{in}$ is introduced as measure for the amplification.

F. Optical resonator

The enhancement of the gain is reached by placing the laser medium within an optical resonator build by the two mirrors M as shown schematically in Fig. 7. The incoming light wave is focused by the lenses L in order to enhance the intensity. In dependent on the reflectivity of the mirrors M the light wave passes by $1/(1-R)$ times through the laser medium, e.g., with a reflectivity of $R = 0.95$ an enhancement by a factor of 20 is reached with the optical cavity.

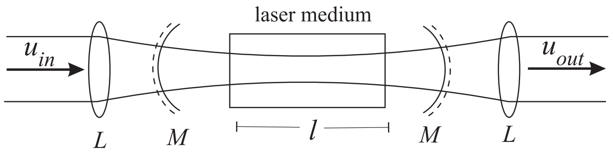

FIG. 7: Optical resonator by two mirrors M with the laser medium. The incoming light wave is focused to enhance the incoming intensity.

G. Pump processes

- Optical pumping. Absorption of (laser) light in the laser medium. Typically found in solid state and liquid laser systems.
- Electrical pumping. Gas re-charging in gas- and semiconductor lasers
- Chemical pumping. $A + B \rightarrow AB*$ (AB*: excited molecule) or dissociative: $AB + h\nu \rightarrow A + B*$ (B*: excited atom)

Three configurations are known for optical pumping with lamps shown in Fig. 8: a) helix-configuration, b) eliptic cavity and c) close coupling. For an efficient optical pumping the spectrum of the pump lamp or pump laser has to be matched for the absorption spectrum of the laser medium. Fig. 9a shows the emission spectrum of a Kr-high pressure lamp and 9b the absorption spectra of the laser media Nd:YAG and Nd:Glass. Absorption bands of the Nd-center occur in the near-infrared region at about 800nm and show a broad absorption band when embedded in glass. Here, the exposure to light of the Kr-high pressure lamp will ensure efficient optical pumping, whereas light of a semiconductor laser with $\lambda = 808$ nm is preferable in Nd:YAG.

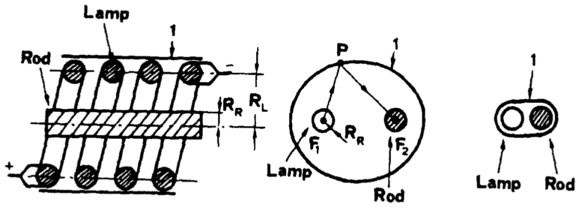

FIG. 8: Optical pumping with lamps a) helix-configuration, b) eliptic cavity and c) close coupling.

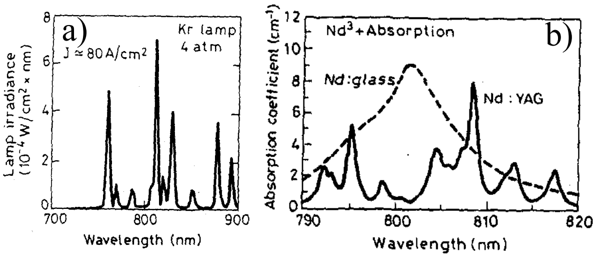

FIG. 9: a) Emission spectrum of a Kr-high pressure lamp, b) Absorption spectra of the laser media Nd:YAG and Nd:Glass.

II. CAVITY DESIGN

A. optical resonator

In the following we will focus on a 4-level laser system of a Nd:YAG laser medium which is optically pumping by a semiconductor laser. Such systems are widely used and commonly called diode pumped solid state laser. A typical scheme of a cavity design is shown in Fig. 10. The divergent light of a Ga-Al-As-semiconductor laser ($\lambda = 808$ nm)is focused via a lens into the Nd:YAG laser rod. As a remarkable feature the optical cavity is realized by dielectric mirrors coated onto the entrance surfaces of the laser rod. A difference of reflectivity of 99.9 % and 99.8 % ensures high and low reflector properties such that emission of laser light occurs into a preferred direction. According to the energetic scheme of the Nd:YAG 4-level system light of wavelength $\lambda = 1064$ nm is emitted. Typical system specifications are a pump power of 1 - 2 W and infrared light of several 100 mWs.

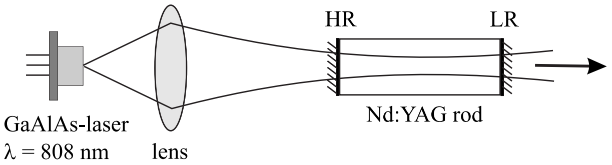

FIG. 10: Schematic setup of a diode pumped Nd:YAG laser cavity. The divergent light of a Ga-Al-As-semiconductor laser ($\lambda = 808$ nm) is focused via a lens into the Nd:YAG laser rod. The optical cavity is realized by dielectric mirrors coated onto the entrance surfaces of the laser rod with different reflectivity.

It is noteworthy that this cavity design enforces high demands to the polishing of the laser rod and to the parallelism of the two entrance surfaces to each other. Other possibilities for a compact cavity design are the prism and spherical resonator (confocal as well as concentric) as shown in Fig. 11a and 11b. Open resonators are of advantage to get linearly polarized light. E.g. in Fig. 11c the entrance faces of the laser rod are cut considering Brewsters law. Internal reflections are suppressed in the in-line configuration by dielectrically coated surfaces (11d). Further, it is possible to influence the laser light by e.g. diaphragm, modulators, filters, optical switches.

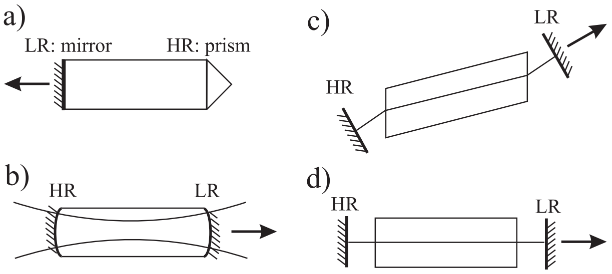

FIG. 11:

B. laser medium

Great demands are additionally required from the Nd:YAG laser rod itself. Beyond the most important are:

- high optical quality: no striations, high optical homogeneity in refractive index and absorption, perfect surfaces
- high optical damage threshold: e.g. cw-laser light up to 1 kW IR at a diameter of 100 μm.
- high conversion efficiency: Nd:YAG e.g. 1-2 %
- high heat flow in order to avoid thermal lens effects
- good preparation and growth conditions in order to get high quality and to reduce costs

C. Losses

An important aspect of cavity design is the balance between light amplification Γ and losses L. For an efficient laser process the condition $\Gamma > l$ has to be fulfilled with the threshold condition $\Gamma - L = 0$. Losses are distinguished from a) the laser rod

- scattering in the volume or on the surface of the laser rod
- absorption in the volume of the laser rod
- reflection losses at laser rod entrance faces
- beam distortion due to refraction or diffraction processes at refractive index inhomogeneities

and b) the laser cavity

- reflection losses and scattering at the mirrors
- absorption losses in the surrounding medium
- coupled-out intensity
- filters, switches, modulators, diaphragm

D. Dimensions of the laser rod

The dimensions of the laser rod, i.e., the length l and the diameter $d = 2 \cdot r$ with $l >> r$, are connected by the Fresnel number

$$F = \frac{n \cdot r^2}{\lambda \cdot l} \tag{6}$$

In order to reduce losses by diffraction the condition $F >> 1$ has to be fulfilled. Typical values are $5 < l < 20.0$ mm. On the other hand the volume of the rod is decisive for the efficiency of optical pumping, which is described by the Schawlow-Townersch relation for a 4-level system:

$$P = \frac{P_0}{P_{(\Gamma - L = 0)}} \cdot \frac{V}{B_{12} \cdot \tau_{21} \cdot \tau_c} \tag{7}$$

whereby τ_c denotes the lifetime of the photons in the laser cavity and P_0 the pump power. Typical values of $\zeta = P_0 / P_{(\Gamma - L = 0)}$ are ~ 1000 for Nd:YAG and ~ 30 for ruby (3-level system).

E. Estimation of the cavity parameter τ_c

The measure τ_c is strongly dependent on the cavity losses and of importance a) to determine laser losses in order to optimize cavity design and b) to determine the optimum pomp power. However, τ_c can not be measured inside the laser cavity. Here, a widely used experimental procedure is to excite the laser process with a single light pulse for optical pumping and subsequent detection of the kinetics of the outcoupled intensity. τ_c is then determined from the periodicity and the damping of the retrieved signal as described in the following.

Optical pumping with pulsed light leads to a temporal development of the number of atoms N_2 in the energy level E_2 of the Nd:YAG 4-level system and thus of the number of photons Q within the optical cavity. The laser rod contributes via

$$\frac{dN_2}{dt} = P - \frac{B_{12}NQ}{V} - \frac{N}{\tau_{21}} \tag{8}$$

with τ_{21} the characteristic lifetime of the spontaneous emission $N_2 \rightarrow N_1$. The second and third terms of equation (8) account for induced and spontaneous emission, respectively. The temporal development of the number of photons in the cavity follows

$$\frac{dQ}{dt} = \frac{B_{12}NQ}{V} + \frac{N}{M\tau_{21}} - \frac{Q}{\tau_c} \tag{9}$$

and is enlarged by induced and spontaneous emission (1st and 2nd term) and is minimized by the restricted lifetime of photons. The measure M accounts for photons which participate in the eigenmode of the optical cavity. The equation system is solved with the linear approximation:

$$\begin{aligned} N &= N_0 + \epsilon; N_0 = \frac{V}{B_{12}\tau_c} \\ Q &= Q_0 + \eta; Q_0 = M - P\tau_c \end{aligned} \tag{10}$$

where ϵ and η are small fluctuations of N_0 and Q_0. Here, the power P inside the laser cavity and the pump power are connected by $P = \zeta \cdot P_0 = \zeta \cdot N_0/\tau_{21}$. Solution of (8) and (9) yields:

$$\left.\begin{matrix} \eta \\ \tau \end{matrix}\right\} \sim \exp\left(-\frac{\zeta t}{2\tau_{21}}\right) \begin{matrix} \sin \\ \cos \end{matrix} \sqrt{\frac{\zeta - 1}{\tau_{21}\tau_c}} \tag{11}$$

which represents a harmonic oscillation with period

$$T^2 = 4\pi^2 \frac{\tau_c \tau_{21}}{\zeta - 1} \tag{12}$$

and a damping constant

$$\tau_d = \frac{2\tau_{21}}{\zeta} \tag{13}$$

In the approximation $\zeta \approx 1$ we get : $T^2 = 2\pi^2 \tau_c \tau_d$, so that τ_c can be determined by the periodicity T and the damping constant τ_d of the detected laser intensity.

F. Reduction of unwanted eigenmodes

The elimination of unwanted longitudinal eigenmodes is connected with a mechanical stable cavity, which is realized by a high temperature control of the cavity and all optical elements, including especially the laser rod, and the use of materials with extreme low extension coefficients, such as super invar. Unwanted transversal eigenmodes are suppressed by introducing diaphragm inside the optical cavity. A birefringence filter, i.e., a combination of polarizer and retarder wave-plate, is commonly used to get linear polarized laser light with an extremely small bandwidth. Fig. 12 shows the setup of a high-quality optical resonator with a temperature controlled base plate and laser rod, a birefringence filter BF and a diaphragm D. Note the specific demands for the dielectric coatings of the laser rod (low reflection coating for $\lambda = 1064$ nm and $\lambda = 808$ nm) and for the transmission of the high reflector (high transmission for $\lambda = 808$ nm, high reflection for $\lambda = 1064$ nm). Typical specifications of such laser systems are a single pass power of 20-40 mW of infrared light ($\lambda = 1064$ nm) with a pump power of $P_{(\lambda=808nm)} = 2$W and dimensions of the laser rod of 10 mm length and 3×3 mm^1 surfac area. Optimum cavity design leads to an intra-cavity power of 20 - 50 W and of ≈ 500 mW extra-cavity.

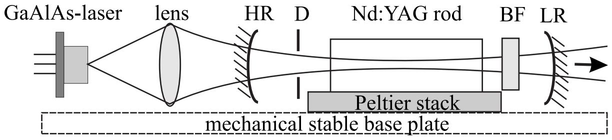

FIG. 12: Setup of a high-quality optical resonator with a temperature controlled base plate and laser rod, a birefringence filter BF and a diaphragm D.

G. Cavity design with intra-cavity second harmonic generation

The next step is the design of a Nd:YAG laser system with intra-cavity second harmonic generation to get intense continuous wave laser light of $\lambda = 532$ nm. The demands the optical cavity with intra-cavity second harmonic generation (SHG) are

- Two independent adjustable beam waists, one localized in the laser rod and one in the non-linear crystal for SHG. The dimensions of the beam waist in the laser rod has to be adapted for the beam waist of the laser light for optical pumping. The beam waist of in the non-linear crystal should be optimized for a high intensity under consideration of the crystal length.

- A high mechanical stability of the optical cavity over a long term.

- Linear polarized laser light.

Further the redesign of the Nd:Yag optical cavity should account for the following aspects

- losses due to the non-linear crystal
- losses due to SHG
- dielectric coatings for the non-linear crystal ($\lambda = 1064$ nm and $\lambda = 532$ nm)
- transmission of the low reflector (high transmission at $\lambda = 532$ nm
- refractive index of the non-linear crystal influences beam waist intra-cavity

With respect to these demands and aspects it should be stressed that intra-cavity second harmonic generation is inevitably necessary to get intense continuous wave laser light. The power of the frequency doubled beam is $\sim I_{1064}^2$ and $I_{1064}^{\mathrm{ic}} >> I_{1064}^{\mathrm{ec}}$, where ic and ec denote intra- and extra-cavity, respectively. In contrast, SHG with pulsed laser light is commonly realized in an extra-cavity configuration.

The disadvantage of intra-cavity SHG is two-folded: a) a more complicated design of the optical cavity is enforced and an exchange of the non-linear crystal is impossible, e.g., for purposes of optimization, b) the demands to the non-linear crystal are enormous especially due to the extremely high power of the fundamental wave (high risk for optically induced mechanical damage).

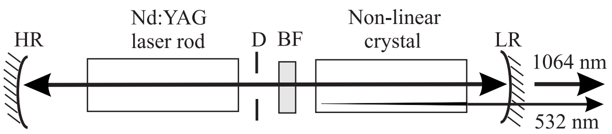

FIG. 13: principle setup for an optical cavity with intra-cavity SHG.

Fig. 13 shows the principle setup for an optical cavity with intra-cavity SHG. All laser properties are restricted for the generation of infrared light at $\lambda = 1064$ nm, i.e., the cavity does not amplify light of $\lambda = 532$ nm. The emission of the frequency doubled laser light occurs in both directions, but is blocked by the polarizer of the birefringence filter (the orientation of the electric field vector for type I and type II phase matching are different to the electric field vector of the fundamental wave). The intensity of the visible light is comparable small in such systems, e.g., with a pump power of 2 W and an intra-cavity power of 10 - 50 W a laser beam with ≈ 150 mW at $\lambda = 532$ nm is generated in the output.

H. Losses by the non-linear crystal

As already mentioned intra-cavity SHG represents an additional loss and thus the demand for a large SHG coefficient is questionable. The condition for the threshold of the laser process with intra-cavity SHG now follows the connection: $G - L - K \cdot P_{1064}$ with $P_{532} = K \cdot P_{1064}^2$ and the non-linear coupling coefficient

$$K = K_{IR} \cdot l_i \cdot k_{IR} \cdot h(\sigma, \zeta) \cdot 10^7 \tag{14}$$

Here, $k_{IR} = 2\pi n_{IR}/\lambda_{IR}$ denotes the wave vector of the infrared laser beam, l_i the interaction length of the fundamental and harmonic waves and K_{IR} is a material specific constant, e.g., $K_{IR} = 128\pi^2\omega^2/c^3 n_{IR}^2 n_{VIS} \cdot d_{32}$ for $Ba_2NaNb_5O_{15}$. The function $h(\sigma, \zeta)$ from the theory of Boyd and Kleinmann takes diffraction, double refraction and absorption processes into account, whereby $\sigma = 1/2b\Delta K$ is connected to the phase matching parameter ΔK and $\zeta = l_i/b$ to the confocal parameter $b = \omega_0^2/k_{IR}$ with the beam waist ω_0. The dependencies of P_{SHG} on the coupling coefficient and of the coupling coefficient on the beam waist are shown in Fig. 14.

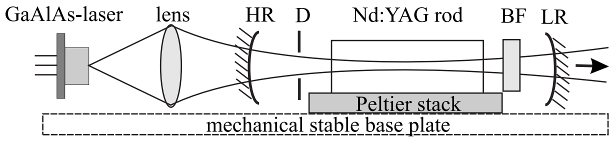

FIG. 14: P_{SHG} as a function of the coupling coefficient K and of K on the beam waist ω_0

I. Selection of the non-linear crystal

The selection of an adequate non-linear crystal is restricted by

- large SHG coefficient
- refractive indizes and disperions
- optical transmission range
- phase matching properties
- optical damage threshold
- optically induced mechanical damage threshold
- optical homogeneity un refractive index and absorption coefficient
- hardness, chemical stability.

Some of the commonly used non-linear crystals are listed in tabular I.

$KTiOPO_4$ is widely used for intra-cavity second harmonic generation of cw-laser light. An important feature is its pronounced birefringence, which is used in combination with a polarizer as birefringence filter.

	transparency range (nm)	damage threshold (GW/cm^2)	FOM
β-BaB$_2$O$_4$	198-3300	10	15
Ba$_2$NaNb$_5$O$_1$5		0.001	
KH$_2$PO$_4$	200-1500	0.5	1
LiB$_3$O$_5$		2	1
LiNb=$_3$		0.02	
LiIO$_3$	300-5500	0.05	50
KTiOPO$_4$	350-4500	1	215
KNbO$_3$	410-5000	0.35	1755
CsD$_2$AsO$_4$	1660-2700	0.5	1.7
(NH)$_2$CO	210-1400	1.5	10.6
LAP	220-1950	10	40
m-NA	500-2000	0.2	60
MGO-LiNbO$_3$	400-5000	0.05	105
POM	414-2000	2	350
MAP	472-2000	3	1600
COANP	480-2000		4690
DAN	430-2000		5090
PPLiNbO$_3$	400-5000	0.05	2460·N^2

TABLE I: Properties of non-linear crystals. FOM is determined by $(d^2/n^3)(EL/\lambda)\Delta\theta^2$. LAP: L-arginine phosphate monohydrate, m-NA: meta nitroaniline, POM: 3-methyl-4-nitropyridine N-Oxide, MAP: methyl (2,4-diinitrophenyl) aminopropanoate, COANP: 2N-cyclooctylamino-5-nitropyridine

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